

A DEFINITION OF GAS-LIQUID PLUG FLOW IN HORIZONTAL PIPES

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Abstract—Experiments were performed to define the transition from slug flow to plug flow in a horizontal pipe. In slug flow the front of the gas cavity is a “Benjamin bubble” and the back is a hydraulic jump. At a Froude number $[(C - u_{L1})/(gD)^{0.5}] \leq 2$, a transition is defined whereby the liquid increases in height in stages in the back of the cavity. For low gas throughputs, it is shown that the front of the cavity can still be represented by the Benjamin bubble.

Key Words: horizontal two-phase flow, slugs, plugs, flow regimes

INTRODUCTION

The intermittent gas-liquid flow pattern observed in horizontal pipes is commonly subdivided into two subregimes—slug flow and plug (elongated bubble) flow. This paper presents the results of experiments aimed at defining how slug flow changes to plug flow with decreasing gas velocity. Most studies on intermittent flows have dealt with slugs, so the characteristics of the plug flow pattern are not well-defined.

This study is largely based on a previous paper by Ruder *et al.* (1989) [which is a further development of ideas presented by Dukler & Hubbard (1975)] that was aimed at establishing necessary conditions for the existence of slugs. From a photographic study, Ruder *et al.* (1989) defined the front of an idealized unaerated liquid slug as a sudden expansion and the back as the front of a “Benjamin bubble” (Benjamin 1968). From these observations two conditions were established.

The first of these considers the height of the liquid carpet that exists for high-frequency stable slugs. A decrease in the height of the liquid in the stratified flow in front of a slug results in a decrease in the rate at which the hydraulic jump accumulates liquid, q_F . Slug stability requires that q_F is not less than the rate at which liquid is shed from the tail, q_T . The condition $q_F = q_T$ establishes a minimum carpet height, h_{\min} (which is less than the height required for the initiation of slugs from the growth of small disturbances). The ratio of h_{\min} to the pipe diameter, D , was suggested to be a function of a Froude number, defined as

$$Fr = \frac{C_F - u_1}{(gD)^{0.5}}, \quad [1]$$

where C_F is the translational velocity of the front of the slug and u_1 is the velocity of the liquid in the carpet. The description of the tail as a Benjamin bubble puts restraints on C_F for a stable slug so that the following relation is obtained:

$$Fr = 0.542 \frac{\pi D^2}{4A_{L1}}, \quad [2]$$

when A_{L1} is the cross-sectional area occupied by the liquid carpet and the r.h.s. of [2] is a function of only h_{\min}/D .

A second necessary condition is derived from the definition of the front of the slug as a sudden expansion. It requires that the Fr is larger than the values given in table 1 of Ruder *et al.* (1989).

Measurements of liquid height and C_F presented in Ruder *et al.* (1989) for air-water flowing in a horizontal, 9.53 cm i.d. pipe ($U_{SG} = 0.71-5$ m/s; $U_{SL} = 0.5-0.95$ m/s) confirmed [2] for slugs with

little aeration and gave values of Fr greater than those listed by Ruder *et al.* in their table 1 for all situations in which slugs were observed.

The present work was done for $U_{SG} < 0.71$ m/s to find out if the same necessary conditions for the front and tail of a slug could be applied to the situations where the gas flow decreases and slugs, consequently, deteriorate.

Measurements of pressure pulsations, void fraction and photographic studies were explored as a means of defining the transition from slug flow to plug flow, where a slug is defined as a liquid blockage propagating down the pipe whose front may be approximated by a single-stage hydraulic jump.

EXPERIMENTS

The experiments were carried out in equipment described in Ruder *et al.* (1989). Water and air flowed through a horizontal pipe of i.d. 0.0953 m and length 24.6 m. The entry was a T-junction in which the liquid flowed in the straight run and the air entered from the top. Liquid heights and slug (or plug) velocities were measured with two conductance probes, the first of which was located $190D$ from the entrance. The probes were separated by a distance of 0.2667 m. Each probe was constructed with two parallel stainless steel wires, 1.5 mm dia and separated by a distance of 19.05 mm, that were strung through the cross section of the pipe. The pressure relative to the atmosphere was measured with a differential pressure transducer (Sensotec, 1EA A5/822-07) with a full pressure range of 0 ± 0.14 bar. The pressure taps were located on the bottom of the pipe so as to avoid, as much as possible, the presence of air in the lines leading to the transducer. The pressure transducer was connected, through an amplifier, to an LSI-11 minicomputer where the voltage signals were registered and stored, along with those from the probes, for later analysis.

Studies were made for conditions ($U_{SG} = 0.018$ – 0.62 m/s and $U_{SL} = 0.5$ – 0.95 m/s) that correspond to plug flows in the empirical flow map presented by Mandhane *et al.* (1974).

PHOTOGRAPHS

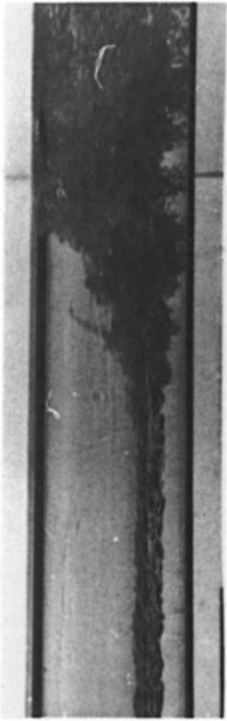
In describing the transition from slug flow it is convenient to focus upon the shape of the air pocket.

Photographs of the backs of air pockets are given in figures 1(a–d). The conditions for figure 1(a) are such that $Fr \cong 2.53$. They represent a slug flow for which there is a sudden expansion from a well-defined liquid carpet which is pictured as a hydraulic jump spanning the entire pipe cross section.

As superficial gas velocities decreased below 0.62 m/s ($Fr \leq 2$) the tail was observed to deteriorate and to have staircase shape, typically with two stages, as shown in figure 1(b). The second stage, which could be longer than 2 pipe diameters, typically reached the top of the pipe. It is to be noted that the liquid in both stages is slightly aerated. A close examination of the photographs of the liquid behind the last stage showed that the top of the pipe was lubricated with a thin air film that is dragged along by the liquid. With a further decrease in U_{SG} the level of the liquid in front of the first hydraulic jump increased and the height of the second stage decreased. At $U_{SG} \cong 0.1$ m/s ($Fr \cong 1.2$ – 1.5) the tail of the gas pocket has the same appearance as the front. It took the form of a gas bubble with slightly aerated liquid behind it as shown in figure 1(c).

It is noted in figures 1(a–c) that the stratified flow in front of the tail was approximately of uniform height. It appears that this liquid is governed by the drag of the gas flow. At $U_{SG} \leq 0.1$ m/s ($Fr \cong 1.2$ – 1.5), the liquid layer appeared no longer to be uniform. The liquid in front of the tail was moved along both by hydraulic gradients and by gas drag. The bubble-shaped tail then deteriorated to the configuration shown in figure 1(d).

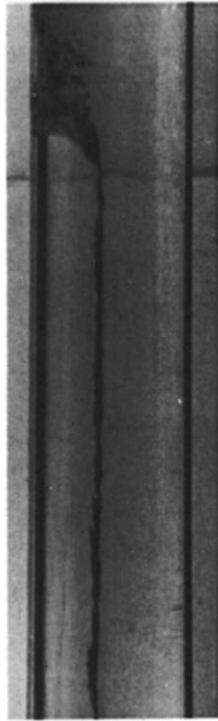
Photographs of the fronts of air pockets are shown in figures 2(a–c). It was found that they had the shape of an air bubble moving through a liquid with $h/D \cong 0.56$; they had the same appearance as the backs of liquid slugs, as described in Ruder *et al.* (1989). Interestingly, the characteristic feature of the very rapid decrease in the liquid height in the front region of the bubble has been observed for all gas velocities tested, both in Ruder *et al.* (1989) and in the present study. This would further imply that inertia forces dominate this region and, therefore, that the steady-state,



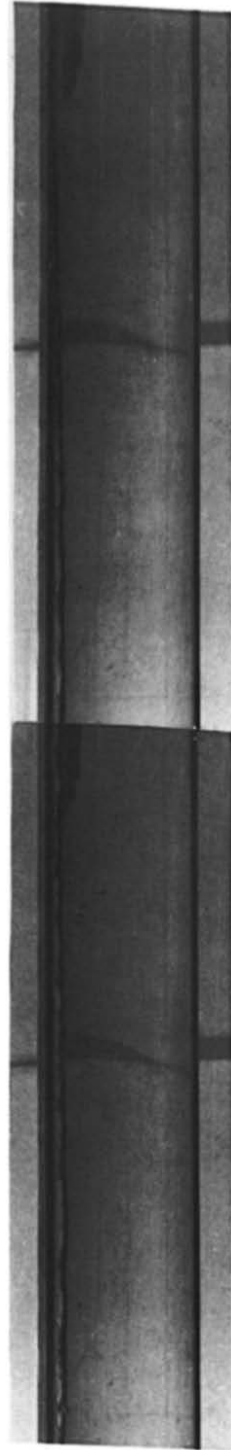
(a) Typical back of a gas cavity in the slug flow ($U_{SG} = 1.45$ m/s; $U_{SL} = 0.95$ m/s; $Fr = 2.53$).



(b) Typical two-stage, staircase-like back of a gas cavity in the plug flow ($U_{SG} = 0.6$ m/s; $U_{SL} = 0.95$ m/s; $Fr = 1.8$).

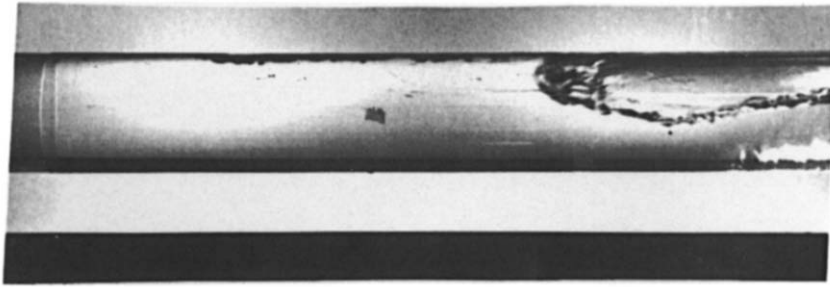


(c) The back of a symmetrical gas cavity in the plug flow ("Benjamin bubble") ($U_{SG} = 0.1$ m/s; $U_{SL} = 0.95$ m/s; $Fr = 1.2$).

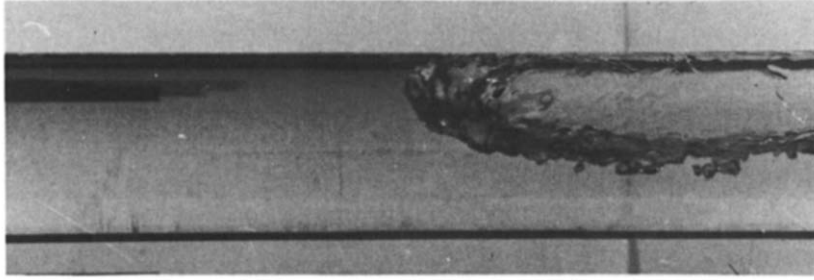


(d) Hydraulic gradient-affected gas cavity at very low gas throughputs ($U_{SG} = 0.018$ m/s; $U_{SL} = 0.95$ m/s).

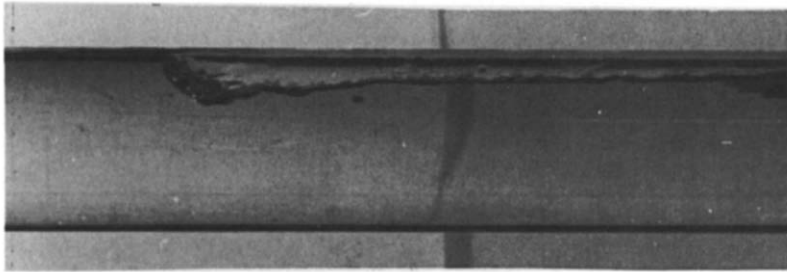
Figures 1(a-d). Photographs of the backs of air pockets.



(a) Typical front of a gas cavity in the slug flow ($U_{SG} = 1.45$ m/s; $U_{SL} = 0.3$ m/s).



(b) Typical front of a gas cavity in the plug flow ($U_{SG} = 0.5$ m/s; $U_{SL} = 0.95$ m/s).



(c) Typical front of a gas cavity in the plug flow at very low gas throughputs ($U_{SG} = 0.018$ m/s; $U_{SL} = 0.95$ m/s).

Figures 2(a–d). Photographs of the fronts of air pockets.

inviscid flow assumption of Benjamin could be applicable to the cases considered. For gas velocities < 0.1 m/s the front of the pocket still had a bubble shape as shown in figure 2(c), but the liquid level surrounding the bubble was higher.

From these photographic studies two definitions of a transition to plug flow are suggested: the first is the appearance of a staircase hydraulic jump at $Fr \cong 2$. The second is the appearance of a symmetric air pocket with a bubble-shaped front and back at $Fr \cong 1.2$ – 1.5 . It should be noted that for the flow conditions tested in the present study, the aeration of the immediate post-tail region of the bubble, i.e. inside the body of a liquid slug (plug), was observed to be very small.

MEASUREMENTS

Typical measurements with the conductance probes are shown in figure 3. These correspond to conditions where there are many plugs in the pipe. As was explained in Ruder *et al.* (1989), the minimum possible height of the liquid carpet in front of a slug (plug) is measured under such conditions. These minimum heights appeared, for the present conditions, to be very weakly dependent on U_{SL} , a trend which was previously reported for high gas throughputs, i.e. for slug flow (Ruder *et al.* 1989).

The tracing for $U_{SG} = 0.62$ m/s corresponds to a slug flow just prior to the appearance of plug flow. The time response of the conductance probes was such that the tracings do not capture the shape of the back of the air space in figures 2(a, b). These photographs show that, on average, the

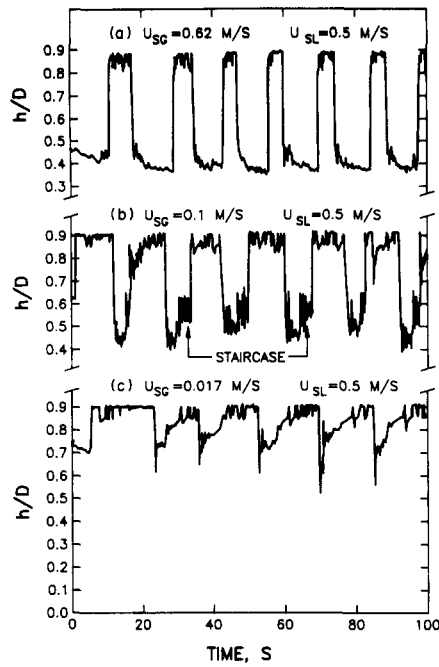


Figure 3. Measurements with conductance probes.

height of the liquid right behind the liquid slug or plug is roughly approximated (for a distance too short to be captured by the conductivity probes) by a value of $h/D = 0.563$ predicted for a Benjamin bubble. The stratified flow between two slugs thins out because of the drag of the gas flow and is roughly fully-developed before the appearance of another liquid slug. Figure 3(b) shows height tracings for a plug flow for which the tail of the gas pocket has the staircase appearance, shown in figure 2(b). Figure 3(c) shows a tracing for a very low U_{SG} for which the tail of the gas pocket has completely deteriorated [as shown in figures 1(d) and 2(c)] and there are large hydraulic gradients in the liquid below the gas pocket.

It is noted in all of the tracings in figures 3(a-c) that the maximum h/D is closer to 0.9 than to 1.0. For the conditions of low aeration tested, this could be associated with the observations in the photographs of a thin gas layer close to the top of the pipe. This layer does not appear to be as large as 0.10, but the sensitivity of the probes to changes in h/D in this range is poor and

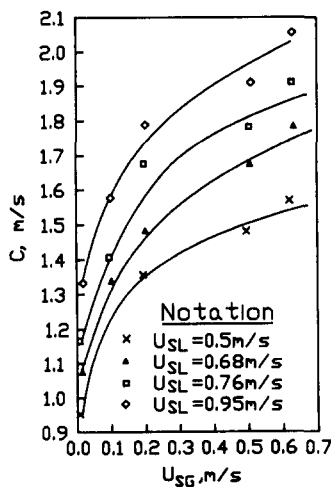


Figure 4. Measurements of the velocity of the air pocket.

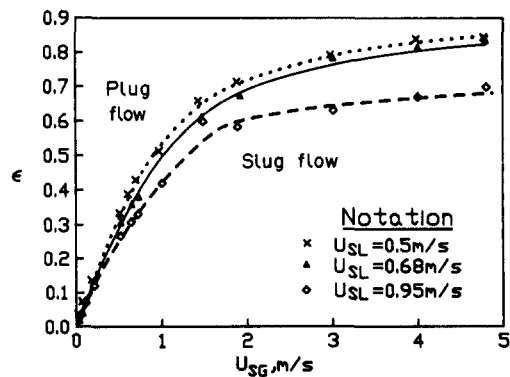


Figure 5. Void fractions calculated from the measurements in figure 4 as $\epsilon = C/U_{SG}$.

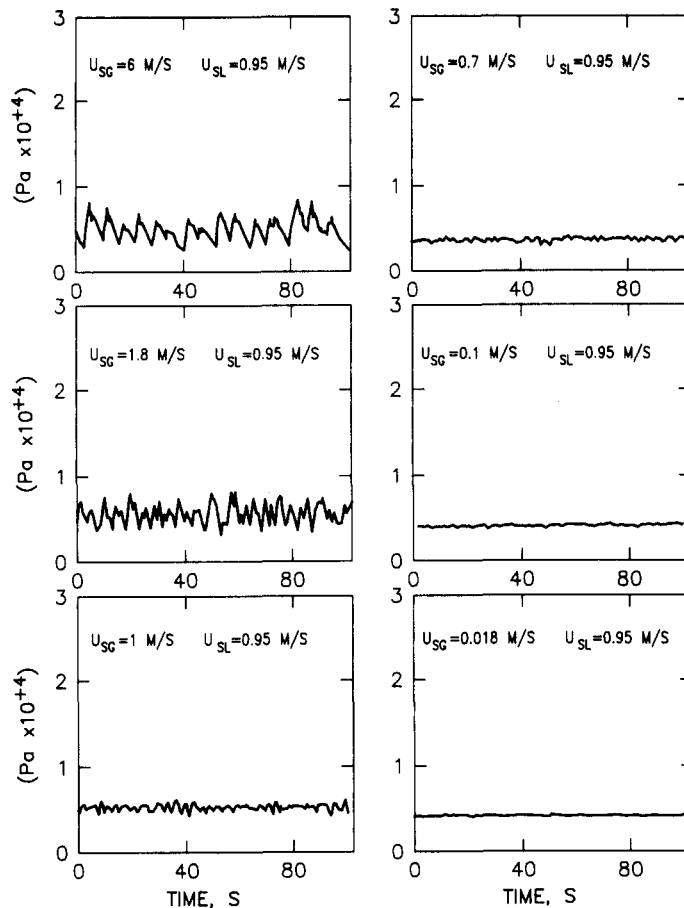


Figure 6. Measurements of pressure fluctuations.

the height of the air layer at the center of the pipe could be larger than what is observed photographically at the wall.

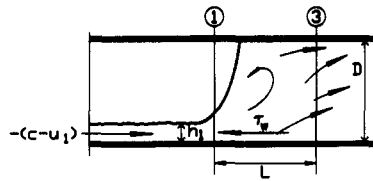
Measurements of the plug velocity, C , are given in figure 4. Since very little of the flowing air is entrapped in the liquid, C should be close to the gas velocity, U . Values of the void fraction calculated from the measurements in figure 4 as $\epsilon = C/U_{SG}$ are plotted in figure 5. An interesting feature is that a much more rapid change of ϵ with decreasing U_{SG} is obtained for plug flow than for slug flow.

Measurements of the pressure fluctuations as a function of superficial gas velocity, presented in figure 6, show that a decrease in U_{SG} is accompanied by a gradual decrease in the amplitude of the pressure fluctuations. These are difficult to interpret; the reason for inclusion is because other investigators (Weisman *et al.* 1979; Lin & Hanratty 1987; Damianides & Westwater 1988) have used them to detect flow patterns. The most important result is that a clearcut definition of the transition from slug to plug flow is not apparent in these experiments in a 9.53 cm pipe. Photographic studies provides a better criterion than measurements with pressure transducers or with conductance probes.

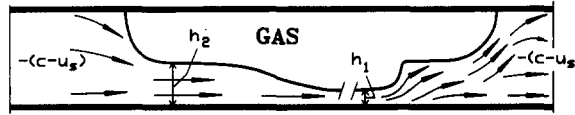
THEORETICAL CONSIDERATIONS

(a) Definition of the slug-plug transition

Following the visual observations discussed above, a definition of the transition to plug flow is adopted which pictures the back of the gas pocket to change from the hydraulic jump shown in figure 1(a) to the staircase pattern shown in figure 1(b). It is depicted in figures 7(a, b) as a transition from the pattern shown in (a) to the pattern in (b). The plateau of height h_2 at the back of the



(a) Schematic of the front of a slug.



(b) Schematic of a gas cavity in plug flow.

Figures 7(a, b). Schematic definition of the transition from slug to plug flow.

gas pocket is somewhat exaggerated. The broken lines are shown to emphasize that the length of the carpet of height h_1 is much larger than indicated in figure 7(b). The velocities are indicated relative to a frame of reference moving with the velocity of the gas pocket, C .

(b) Model for the front of the gas pocket

The front of the gas pocket was approximated for slug flow as a Benjamin bubble by Ruder *et al.* (1989). The photographs suggest that the same model might be valid for plug flows. According to the solution presented by Benjamin (1968),

$$\frac{C - u_s}{(gD)^{0.5}} = 0.542 \tag{3}$$

and

$$\frac{h_2}{D} = 0.563, \tag{4}$$

where u_s is the velocity of the liquid in the slug and h_2 is defined in figure 7(b).

It is noted that the Fr in [3] differs from [1] in that liquid velocity u_s is used rather than u_1 . In order to test [3] it is necessary to evaluate u_s , which is less than the bubble velocity, C , or the gas velocity, U . Two methods are used.

For large gas velocities, where a well-defined carpet of height h_1 exists, the approach is the same as used by Ruder *et al.* (1989) for slug flow. From conservation of mass:

$$C - u_s = (C - u_1) \frac{A_{L1} 4}{\pi D^2}, \tag{5}$$

where the velocity in the carpet, u_1 , becomes increasingly smaller than C and u_s as the gas velocity increases. Velocity u_1 is calculated from measurements of h_1 by using the stratified flow model developed by a number of researchers (Taitel & Dukler 1976; Andritsos & Hanratty 1987). For $U_{SG} < 0.2$ m/s, hydraulic gradients exist at the gas-liquid interface [see figures 2(c) and 1(d)] and u_1 cannot be estimated from relations for fully-developed stratified flows. The velocity u_s was roughly approximated in these cases from a relation which assumes that u_s is not much different from the average velocity of the liquid in the pipe:

$$u_s \cong \frac{U_{LS}}{(1 - \epsilon)}. \tag{6}$$

(c) The tail of the gas pocket

The tail of the gas pocket shown in figure 7(b) is approximated as a one-step sudden expansion from h_1 to $h_3 = D$. The velocities of the fluid at 1 and 3 are assumed uniform and the fluid at 1 and at 3 are assumed to be unaerated. A momentum balance between 1 and 3 yields

$$\rho(C - u_3)^2 A_{L3} - \rho(C - u_1)^2 A_{L1} = (p_1 - p_3) A_{L3} + \rho g h_{PC1} A_{L1} - \rho g h_{PC3} A_{L3} - F_w, \tag{7}$$

where h_{PC} is the pressure centroid and F_w is the resisting force exerted by the pipe walls. For a hydraulic jump just on the verge of deterioration, the pressure force term in [7] can be omitted. Then

$$(C - u_1)^2 \left(\frac{A_{L1}}{A_{L3}} \right)^2 - (C - u_1)^2 \frac{A_{L1}}{A_{L3}} - gh_{PC1} \frac{A_{L1}}{A_{L3}} + gh_{PC3} + \frac{F_w}{\rho A_{L3}} = 0, \quad [8]$$

where conservation of mass

$$(C - u_3) = (C - u_1) \frac{A_{L1}}{A_{L3}} \quad [9]$$

has been used to eliminate u_3 . The case where the momentum flow in the carpet is large enough for the liquid to just reach the top wall is defined by [8] with $A_{L3} = A = \pi D^2/4$. For this case, [8] can be rearranged to give

$$Fr = \frac{\left(\frac{1}{2} - \frac{h_{PC1}}{D} \frac{A_1}{A} + \frac{F_w}{gD\rho A} \right)^{0.5}}{\left[\frac{A_1}{A} - \left(\frac{A_1}{A} \right)^2 \right]}. \quad [10]$$

Usual practice is to ignore F_w . This condition previously considered by Johnson (1987) and Jepson & Taylor (1988) is very similar to the necessary condition 1 for the existence of slugs presented by Ruder *et al.* (1989). It gives a minimum value of $Fr = (C - u_1)/(gD)^{1/2}$ for the existence of slugs of about 1 which is smaller than the value of $Fr \cong 2$ visually observed for the breakdown of the single-stage jump to a two-stage jump.

A possible explanation for this discrepancy is the ignoring of F_w . In order to assess this effect the wall resistance, τ_w ,

$$F_w = \tau_w \pi DL \quad [11]$$

was approximated by the Blasius equation for turbulent liquid flow in a pipe:

$$\tau_w = 0.046 \left(\frac{D}{v} \right)^{-0.2} \frac{\rho}{2} u_s^{1.8}. \quad [12]$$

In [12] it is assumed that, although the control volume just behind the sudden expansion contains a circulating zone, the liquid velocity distribution at low gas velocities is uniform enough so that the wall stresses act over the entire length, L , from the front of the jump to 3. For flow conditions where the height of the liquid carpet was well-defined ($U_{SG} \geq 0.1$ m/s), the mass balance over the front of a slug (plug) can be used with [12] to obtain

$$\tau_w = 0.046 \frac{\rho}{2} \left(\frac{D}{v} \right)^{-0.2} \left[C \left(1 - \frac{A_{L1}}{A} \right) + u_1 \frac{A_{L1}}{A} \right]^{1.8}. \quad [13]$$

(d) Deterioration of the plug at low U_{SG}

A decrease in U_{SG} was observed to be accompanied by an increase in h_1 , defined in figure 7(a). Eventually a condition is reached for which $h_1 = h_2$. It would be expected that for U_{SG} smaller than this the gas pocket would start to deteriorate. Therefore it is suggested that transition from the condition depicted in figure 1(b) to that depicted in figure 1(d) would occur for

$$Fr = \frac{C - u_1}{(gD)^{0.5}} = 0.542 \frac{\pi D^2}{4A_{L1}}, \quad [14]$$

where $A_{L1}/D^2 = 0.456$ is the value corresponding to $h/D = 0.563$.

COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

In figure 8, [3], for the front of the air pocket, is compared with measurements. It is noted that the measured values of $(C - u_s)/(gD)^{0.5}$ are indeed in approximate agreement with the value of 0.542 given for a Benjamin bubble.

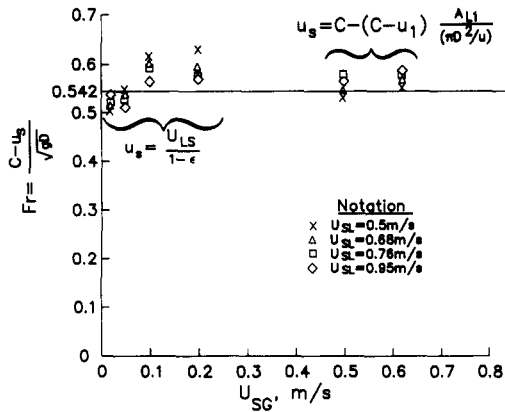


Figure 8. Comparison of measurements of the velocity of the gas pocket with Benjamin's analysis.

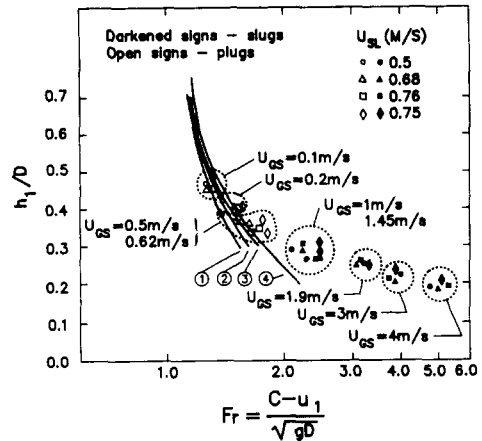


Figure 9. Transition to slug or plug flow. Curve 1 is calculated with $F_w = 0$. Curves 2, 3 and 4 are calculated with $L/D = 2D, 5D$ and $8D$, respectively.

Figure 9 presents values of Fr for the conditions for which bubble tails of the type shown in figures 1(a-c) were observed. These results show that the gas pocket becomes a symmetric bubble at $Fr \cong 1.2$, roughly in agreement with the estimate of $Fr = 0.935$, predicted by Benjamin and given by [14]. A staircase behavior starts to develop at $Fr \cong 2$.

The transition from slug to plug flow is defined as $Fr \cong 2$. The curves given in figure 9 are calculated with [10] for $F_w \neq 0$ using different values of L/D . It is noted that the necessary condition for the existence of slugs based on the classical analysis of a hydraulic jump ($F_w = 0$) gives a lower Fr than what is observed for the transition. As is seen in figure 9, the observed difference could be explained by frictional resistance at the wall, but the comparison presented in figure 9 for $F_w \neq 0$ is not conclusive enough because the estimate of wall drag is speculative. It is, however, of interest to note that the Fr values corresponding to slug flow reported by Ruder *et al.* (1989) (the solid symbols in figure 9) appear to fall to the right of the plug-slug transition suggested. The plug flow results are observed to group in the proximity of the curves for $F_w \neq 0$. The scattering of the h/D data in figure 9, especially for $U_{SG} = 0.5$ and 0.62 m/s may well be ascribed to the errors in averaging of the results of measurements of h_{min} taken from the tracings of the type shown in figures 3(a-c).

DISCUSSION

A number of criteria have been presented in the literature for defining the transition from slug flow to plug flow. From experiments in 1.95 and 2.55 cm pipes, Barnea *et al.* (1980) considered the elongated bubble pattern as a limiting case of slug flow when the liquid slug is free of entrained gas bubbles. Weisman *et al.* (1979), Lin & Hanratty (1987) and Damianides & Westwater (1988) used pressure pulsations to identify slugs. The studies in a 9.53 cm pipeline, described in this paper, did not give a sharp definition of the transition using either of these criteria. Photographic studies of the shape of the interface, as used by Govier & Omer (1962) and Hoogendoorn (1959), proved to be more satisfactory. These provide two possible criteria: (1) the back of the gas cavity assumes a staircase shape; and (2) the back of the cavity assumes a bubble shape similar to its front.

The different criteria used to define the transition are not mutually exclusive. The aeration of the slug is particularly large when a hydraulic jump exists, so the disappearance of the hydraulic jump is associated with a sharp decrease of aeration. Thus, there is general agreement between the transition values given by Barnea *et al.* (1980) based on aeration, and those presented in this paper. Both studies indicate that transition is independent of U_{SL} and occurs at $U_{SG} \cong 0.6$ m/s.

An advantage of using photographic criteria is that they associate the transition with changes in the hydrodynamics which can be modeled to provide mathematical criteria. Both the experiments and the analyses presented in this paper show that an Fr defined as $(U - u_1)/(gD)^{0.5}$ is particularly important. At $Fr < 2$ (or $h_1/D > 0.34$) the hydraulic jump at the back of the gas cavity does not

span the pipe cross section. At $Fr \cong 1.2$ the gas cavity becomes a symmetric Benjamin bubble with a minimum $h_L/D \cong 0.56$.

For practical calculations the gas velocity, U , is calculated as U_{SG}/ϵ , where the void fraction, ϵ , can be estimated from a correlation presented by Hughmark (1962). The liquid velocities, u_1 , can be estimated using the approach of Andritsos & Hanratty (1987) or, at very low velocities, as $U_{SL}/(1 - \epsilon)$.

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